

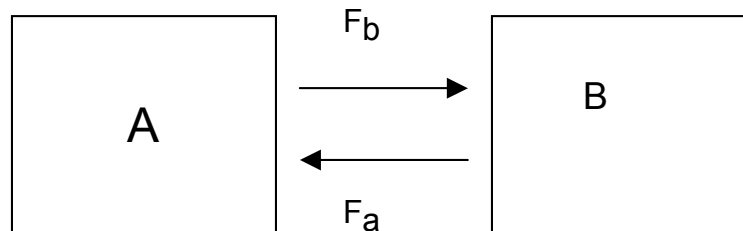
Environmental Geochemistry
Solutions to Practical 4: Open Systems and Kinetics

Solution for problem 1:

Element	Conc. In Rivers (m)	Conc. In Oceans	Residence Time
K	3.4×10^{-5}	1.02×10^{-2}	1.14×10^7
Fe	7.2×10^{-7}	1×10^{-9}	52
Cu	1.6×10^{-7}	4.0×10^{-9}	951
Si	1.9×10^{-4}	1.0×10^{-4}	2.0×10^4

Fe⁺³ and Cu⁺² are hydrolyzed and scavenged. K is inert while Si is taken up by radiolarians.

Solution for problem 2:



$t_a = M_a/F_a$ $t_b = M_b/F_b$ thus $F_a = M_a/t_a$ and $F_b = M_b/t_b$. At steady state,

$F_a = F_b$ or $M_a/t_a = M_b/t_b$. Since, $M_b = M - M_a$ we have

$$M_a/t_a = (M - M_a)/t_b.$$

$$(t_b/t_a)M_a = M - M_a$$

$$(1 + t_b/t_a)M_a = M \quad \text{or} \quad M_a = M/(1 + t_b/t_a)$$

When $t_a \gg t_b$ then $M_a = M$

Solution for Problem 3:

Initially, the global riverine flux of Hg is $Q[\text{Hg}]_{\text{river}} =$

Flux out of the oceans due to scavenging is $Vk[\text{Hg}]_{\text{ocean}}$

At steady state, $F_{\text{in}} - F_{\text{out}} = 0$ so that

$$Q[\text{Hg}]_{\text{riv}} = Vk[\text{Hg}]_{\text{ocean}}$$

Given that the steady state value of $[\text{Hg}]_{\text{ocean}}$ is $5 \times 10^{-12} \text{ m}$ and $[\text{Hg}]_{\text{riv}} = 4 \times 10^{-10} \text{ m}$, we can solve for k to get 0.0021/year.

If we double the riverine concentration we will then double the final steady state value of $[\text{Hg}]_{\text{ocean}}$. How long will this take? Before we reach steady state, we have

$$dM/dt = Q[\text{Hg}]_{\text{riv}} - Vk[\text{Hg}]_{\text{ocean}}$$

$$\text{or } Vd[\text{Hg}]_{\text{ocean}}/dt = Q[\text{Hg}]_{\text{riv}} - Vk[\text{Hg}]_{\text{ocean}}$$

$$\text{or } d[\text{Hg}]_{\text{ocean}}/dt = (Q/V)[\text{Hg}]_{\text{riv}} - k[\text{Hg}]_{\text{ocean}}$$

This is our differential equation. We can solve it by making the substitution

$$X = (Q/V)[\text{Hg}]_{\text{riv}} - k[\text{Hg}]_{\text{ocean}}$$

so that

$$dX = -k(d[\text{Hg}]_{\text{ocean}})$$

Our differential equation becomes

$$(-1/k)dX/dt = X$$

with solution

$$X = X_0 e^{-kt}$$

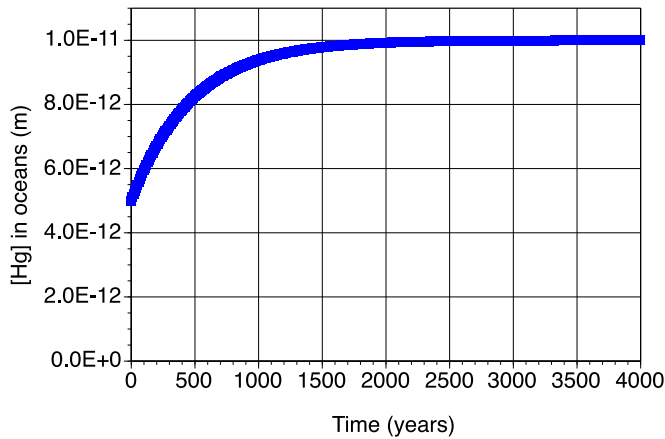
If we substitute back in for X , we get

$$(Q/V)[\text{Hg}]_{\text{riv}} - k[\text{Hg}]_{\text{ocean}} = ((Q/V)[\text{Hg}]_{\text{riv}} - k[\text{Hg}]_{\text{ocean}}^0) e^{-kt}$$

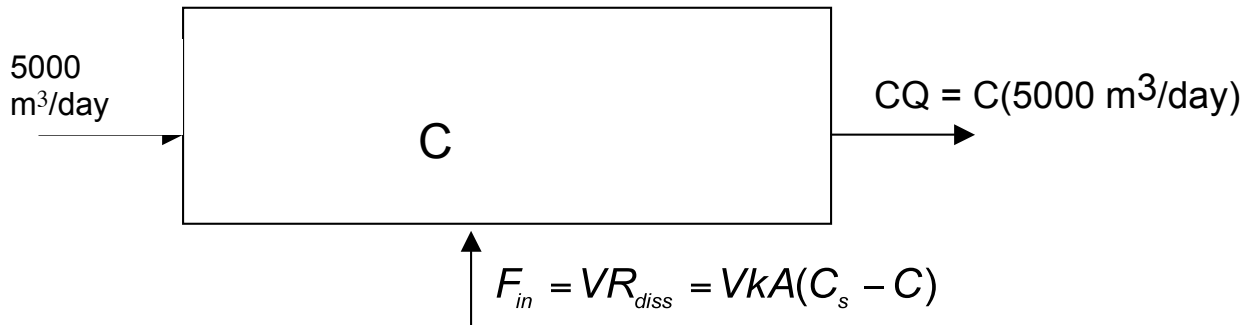
where $[Hg]_{riv}$ is the new river concentration and $[Hg]_{ocean}^0$ is the initial concentration of Hg in the oceans. Hence, if we double the riverine flux, $[Hg]_{riv} = 2[Hg]_{riv}^0$, we can solve for $[Hg]_{ocean}$ as a function of time.

$$[Hg]_{ocean} = (1/k)(Q/V)[Hg]_{riv} - ((1/k)(Q/V)[Hg]_{riv} - [Hg]_{ocean}^0) e^{-kt}$$

$$= 1 \times 10^{-11} + ([Hg]_{ocean}^0 - 1 \times 10^{-11}) e^{-kt}$$



Solution for problem 4:



The mass of Pb in the reservoir is VC . The flux in (by dissolution) is VR_{diss}

$$F_{in} = VR_{diss} = V k A (C_s - C)$$

The flux out is simply $F_{out} = CQ = C (5000 \text{ m}^3/\text{day})$

Conservation of mass (or moles) requires that

$$V(dC/dt) = F_{in} - F_{out} = V[kA(C_s - C)] - QC = 0 \text{ (at steady state)}$$

Hence, we have

$$1000 \text{ m}^3 (0.2 / \text{m}^2\text{-day})(1 \text{ m}^2/\text{m}^3)(15 \text{ mole}/\text{m}^3 - C) = C(5000 \text{ m}^3/\text{day})$$

Solving for C, we get $C = 0.577 \text{ moles}/\text{m}^3$ while the saturation concentration (given) is $15 \text{ mole}/\text{m}^3$.

Solution for problem 5:

At steady state:

$$dM/dt = F_{in} - F_{out} = V(d[C]/dt)$$

$$V(d[C]/Dt) =$$

$$(100 \text{ kg}/\text{day})(25 \text{ mg}/\text{kg}) - (0.48/\text{day})(10000 \text{ kg})[C] - (300 \text{ kg}/\text{day})[C] = 0$$

Solve for [C] to get

$$[C] = 0.49 \text{ mg}/\text{kg}$$